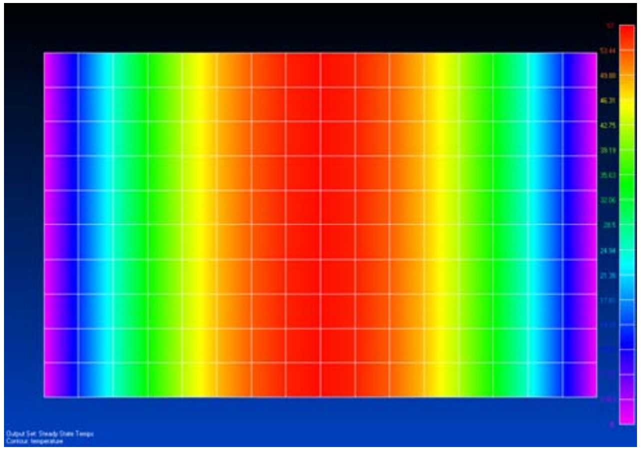
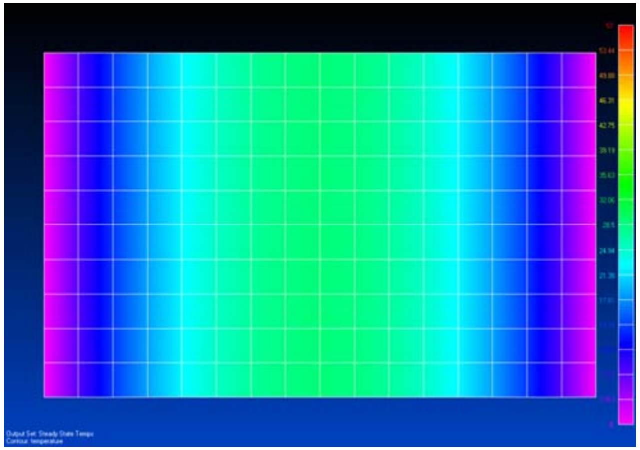
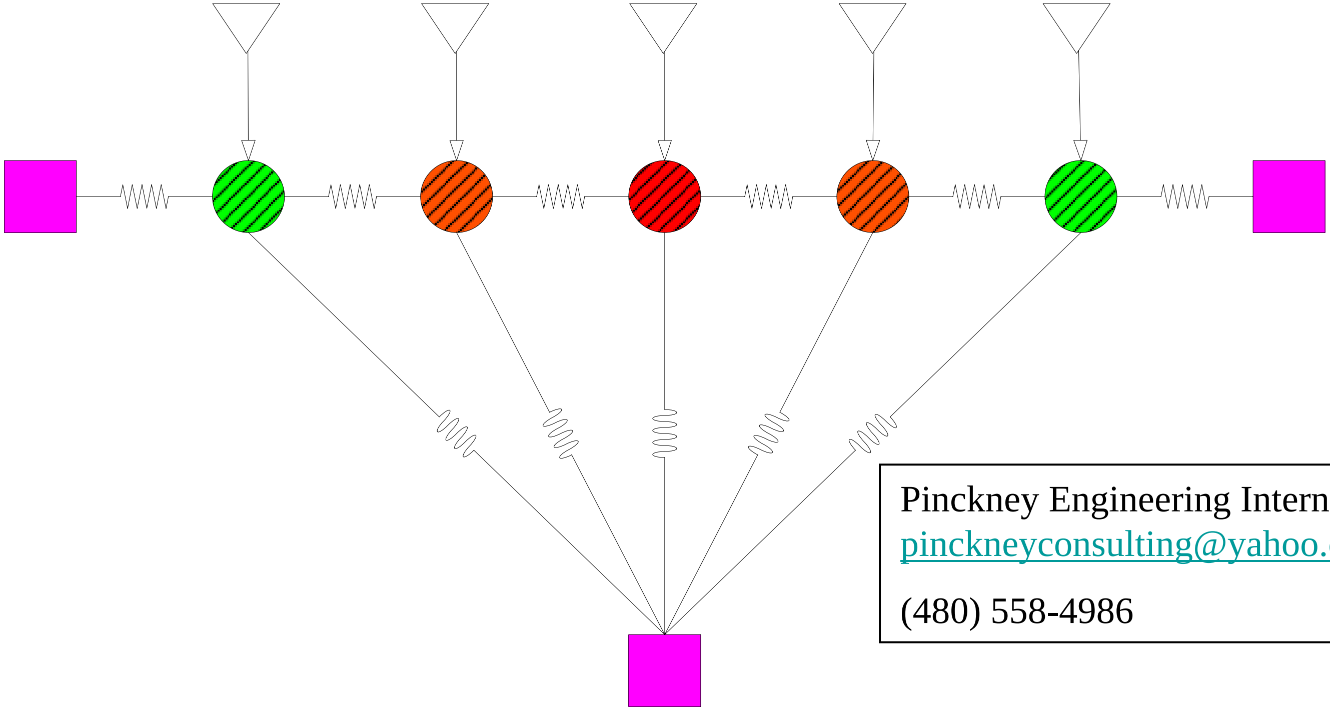
Uniformly Heated Electronics Board







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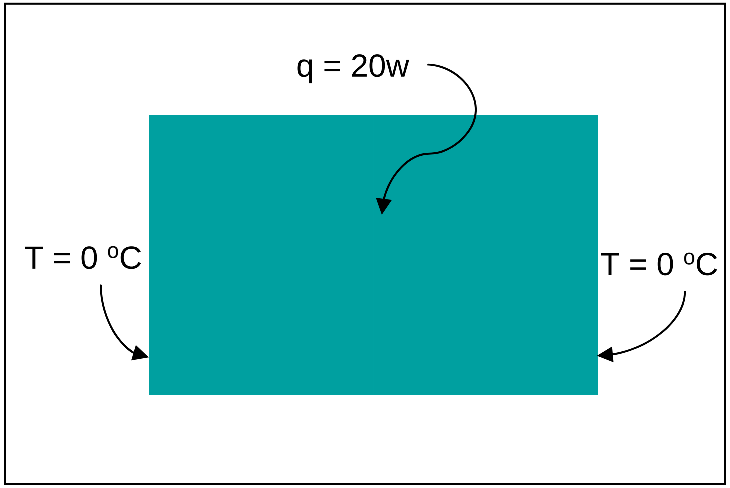
Consider a 5"x8" Electronics Board

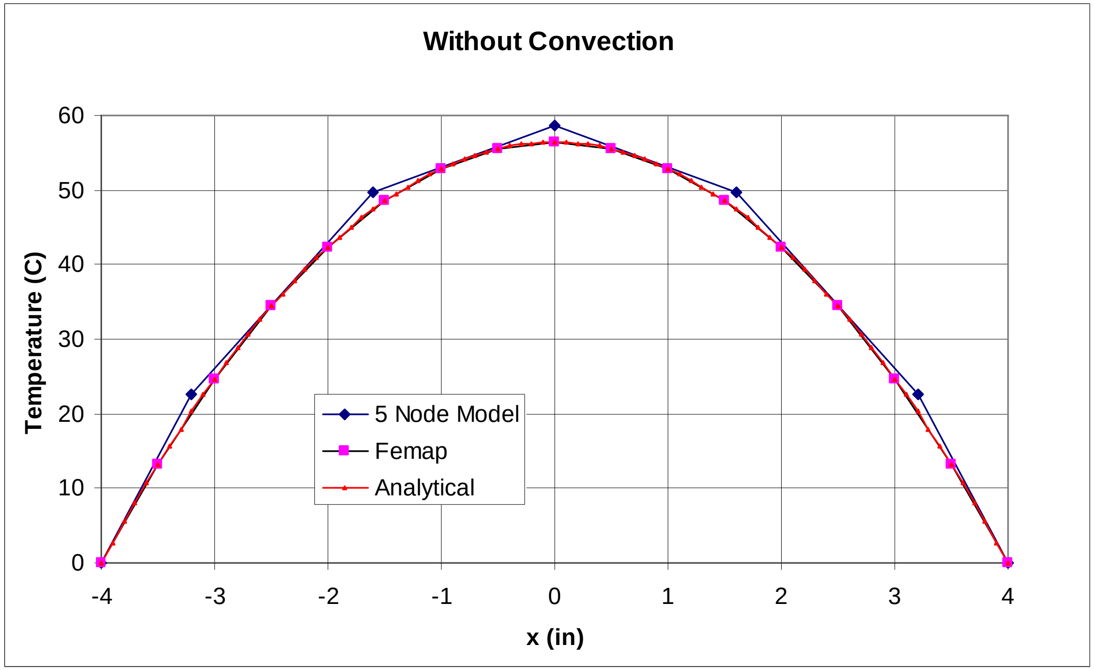
- 20 watts uniformly distributed
- The average thickness of Cu is 0.00716"
- Vertical edges held at T = 0
- Analyze with and without convection
- Compare *Femap* and *SINDA/G* models with analytical results

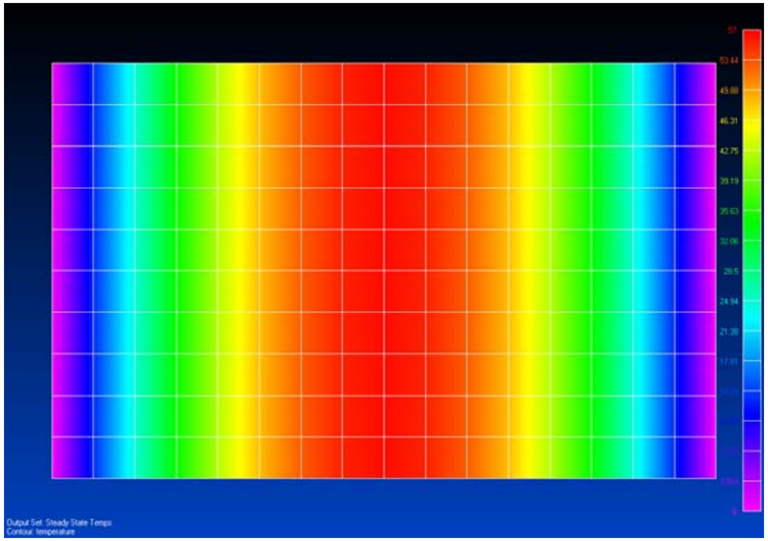


For every project it is advisable to do a simple thermal analysis at the beginning of the design process. Here all the powers of all major components are added, the board sized and the amount of trace material on the board is estimated. Analysis is then performed with the power and trace material uniformly distributed over the board.

(The problem is, of course, symmetric along the vertical center line so all models could be smaller by ½.)

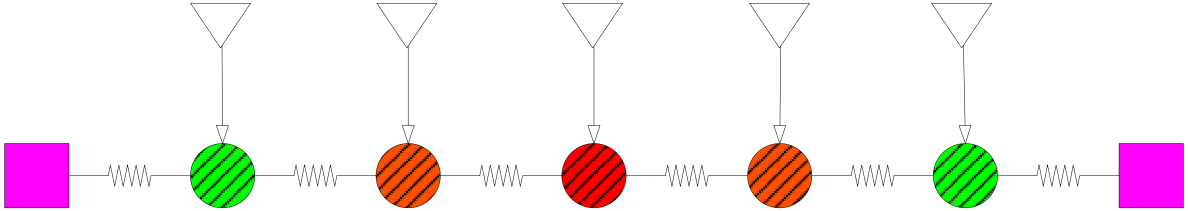
Without Convection





$$T_{noconvection}(x) := \frac{q}{2w\cdot k\cdot t}\cdot\left(\frac{L}{4} - \frac{x^2}{L}\right)$$

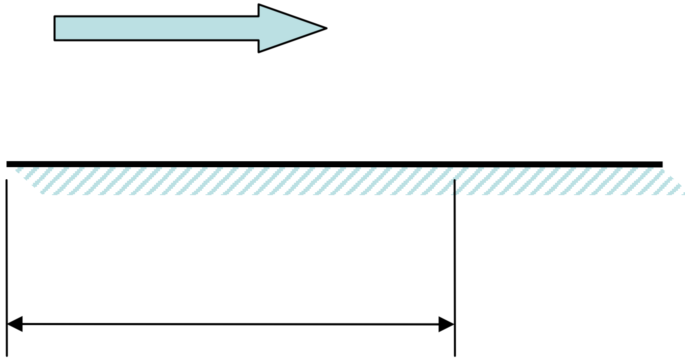
Method	Max T
5 Node	58.69
Femap	56.43
Analytical	56.43



SINDA/G Model

With Convection

Correlation ID = 603



x = distance from leading edge

Consider a standard CPU cooling fan with output of 57 CFM and a cross sectional area of 13.1 in^2.

Air velocity = 3.18 m/sec or 125.2 in/sec

$$h = \frac{k_{fluid}}{x} Nu$$

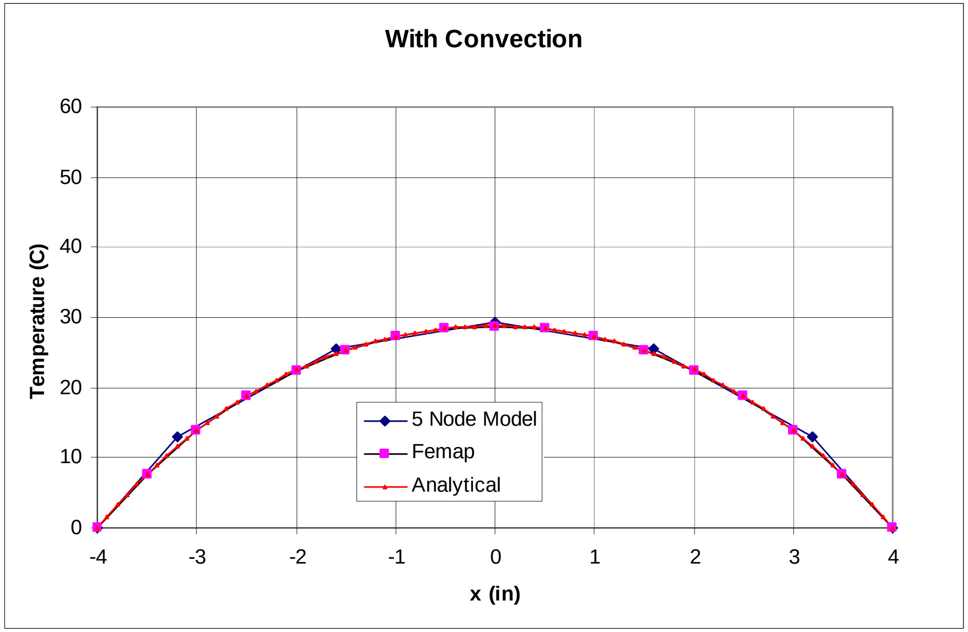
$$where:$$

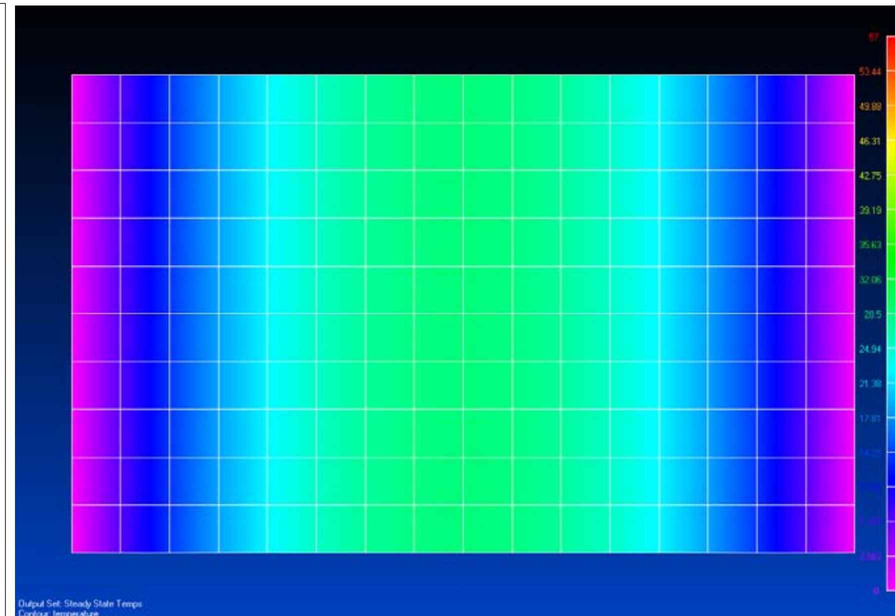
$$Nu = 0.029\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{0.43}$$

$$\mathrm{Re}_x = \frac{Vx\rho_{fluid}}{\mu_{fluid}}$$

Assuming this value for air velocity and uniform flow across the board, SINDA/G correlation 603 gives convection coefficient h = 0.01 W/in^2 C.

With Convection

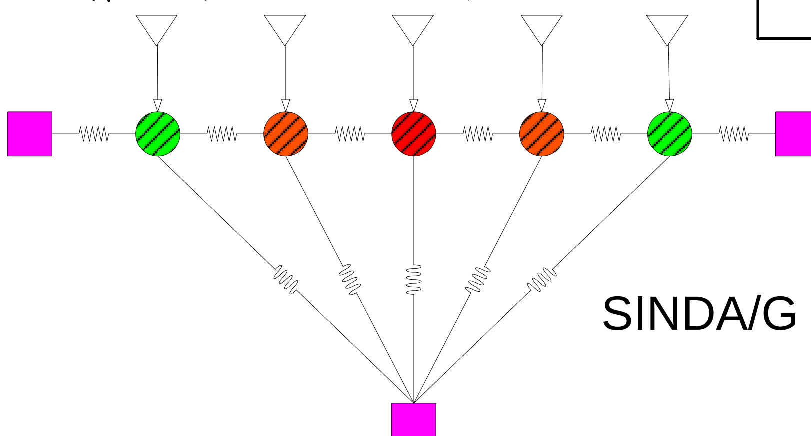




$$T(x) := \left(\frac{q}{L\cdot w\cdot h} + T_a\right)\cdot\left(1 - 2\cdot\frac{\sinh\left(\sqrt{\frac{h}{k\cdot t}}\cdot\frac{L}{2}\right)}{\sinh\left(\sqrt{\frac{h}{k\cdot t}}\cdot L\right)}\cdot\cosh\left(\sqrt{\frac{h}{k\cdot t}}\cdot x\right)\right)$$

$$T_a = 0$$

Method	Max T
5 Node	29.28
Femap	28.74
Analytical	28.79



SINDA/G Model

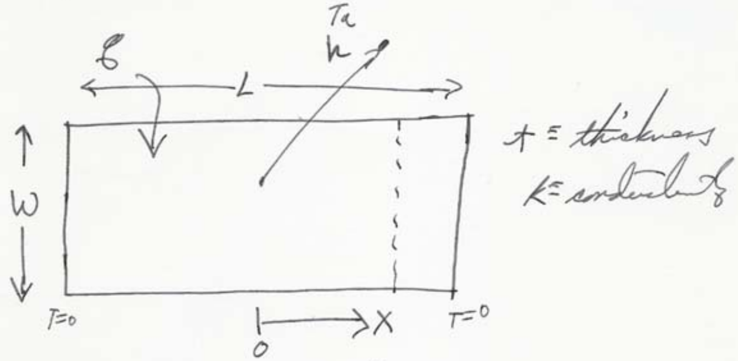
Conclusions

Preliminary study shows:

- The maximum temperature of the board without convection will be 56 ^{o}C above ambient.
- The maximum temperature of the board with convection will be 29 ^{o}C above ambient.
- The three methods, analytical, *Femap* and SINDA/G agree within 4%.
    - It should be noted that Femap and SINDA/G models can be made to agree with the analytical results to within any tolerance by increasing the number of nodes in the models.

Appendix: Derivation of Analytical Result for Uniformly Heated Board with Convection.

Uniformly Heated Board with Convection



$$q'' = \frac{Q}{LW}$$

$$q'' w x - -k w t \frac{dT}{dx} - \int_0^x h w (T(\bar{x}) - T_a)\, d\bar{x} = 0$$

$$\frac{q'' x}{h} + \frac{k}{h} t \frac{dT}{dx} - \int_0^x (T(\bar{x}) - T_a)\, d\bar{x} = 0$$

$$\frac{q''}{h} + \frac{kt}{h} \frac{d^2T}{dx^2} - T + T_a = 0$$

$k = k/h$

$$\frac{q''}{hkt} + \frac{T_a}{kt} = \frac{-d^2T}{dx^2} + \frac{T}{kt}$$

$$\frac{-q''}{hkt} + \frac{-T_a}{kt} = \left(D^2 + \frac{-1}{kt}\right) T$$

$$\left(D^2 - \frac{1}{kt}\right) T = a$$

homo:

$$T_h = c_1 e^{\sqrt{1/kt}\, x} + c_2 e^{-\sqrt{1/kt}\, x}$$

annihilate:

$$D\left(D^2 - 1/kt\right) T = 0$$

$$T_p = c_3$$

$$T = c_1 e^{\sqrt{1/kt}\, x} + c_2 e^{-\sqrt{1/kt}\, x} + c_3$$

Find c_3

$$\left(D^2 - 1/kt\right) T = -\frac{1}{kt}\left(\frac{q''}{h} + T_a\right)$$

$$-\frac{1}{kt} c_3 = -\frac{1}{kt}\left(\frac{q''}{h} + T_a\right)$$

$$c_3 = \frac{q''}{h} + T_a$$

$$T(x) = c_1 e^{\sqrt{1/kt}\, x} + c_2 e^{-\sqrt{1/kt}\, x} + \frac{q''}{h} + T_a$$

apply BC's

$$T(\pm L/2) = 0 = c_1 e^{\sqrt{\ } L/2} + c_2 e^{-\sqrt{\ } L/2} + \frac{q''}{h} + T_a = 0$$

$$= c_1 e^{-\sqrt{\ } L/2} + c_2 e^{\sqrt{\ } L/2} + \frac{q''}{h} + T_a = 0$$

i) $c_1 e^{\sqrt{\ } L} + c_2 + c_3 e^{+\sqrt{\ } L/2} = 0$

ii) $c_1 e^{-\sqrt{\ } L} + c_2 + c_3 e^{-\sqrt{\ } L/2} = 0$

$$c_1\left(e^{\sqrt{\ } L} - e^{-\sqrt{\ } L}\right) + c_3\left(e^{\sqrt{\ } L/2} - e^{-\sqrt{\ } L/2}\right) = 0$$

$$c_1 = -c_3 \frac{\left(e^{\sqrt{\ } L/2} - e^{-\sqrt{\ } L/2}\right)}{e^{\sqrt{\ } L} - e^{-\sqrt{\ } L}}$$

j) $c_1 + c_2 e^{-\sqrt{\ } L} + c_3 e^{-\sqrt{\ } L/2} = 0$

jj) $c_1 + c_2 e^{\sqrt{\ } L} + c_3 e^{\sqrt{\ } L/2} = 0$

$$c_2\left(e^{-\sqrt{\ } L} - e^{\sqrt{\ } L}\right) + c_3\left(e^{-\sqrt{\ } L/2} - e^{\sqrt{\ } L/2}\right) = 0$$

$$c_2 = -c_3 \frac{\left(e^{-\sqrt{\ } L/2} - e^{\sqrt{\ } L/2}\right)}{e^{-\sqrt{\ } L} - e^{\sqrt{\ } L}} = +c_1$$

check:

~~$T(L/2) = \ldots - c_3 (e$~~

$$T(x) = -c_3 \frac{\left(e^{\sqrt{\ } L/2} - e^{\sqrt{\ } L/2}\right)}{e^{\sqrt{\ } L} + - e^{-\sqrt{\ } L}}\left(e^{\sqrt{\ } x} + e^{-\sqrt{\ } x}\right) + c_3 -$$

$T(L/2) = 0$ so $\dfrac{\left(e^{L/2} - e^{-L/2}\right)}{e^{L} - e^{-L}}\left(e^{L/2} + e^{-L/2}\right) = \dfrac{e^{L} + 1 - 1 - e^{-L}}{e^{L} - e^{-L}} = 1$ ✓

(2)

$$T(x) = -\left(\frac{q''}{h} + T_a\right)\left\{\frac{\sinh\sqrt{h/kt}\;L/2}{\sinh\sqrt{h/kt}\;L}\right\}\left(e^{\sqrt{h/kt}\,x} + e^{-\sqrt{h/kt}\,x}\right)$$

$$+ \frac{q''}{h} + T_a$$

$$T(x) = -\left(\frac{q}{Lwh} + T_a\right)\frac{\sinh\sqrt{h/kt}\;L/2}{\sinh\sqrt{h/kt}\;L}\,2\cosh\sqrt{h/kt}\;x + \frac{q}{Lwh} + T_a$$

$$T(x) = \left(\frac{q}{Lwh} + T_a\right)\left\{1 - 2\,\frac{\sinh\left(\sqrt{h/kt}\;L/2\right)\cosh\left(\sqrt{h/kt}\;x\right)}{\sinh\left(\sqrt{h/kt}\;L\right)}\right.$$

$h \to 0$, $T(x) \to$ no convection result